Spin Energy and Rest Mass

We must now turn our attention to the phenomenon of spin in more detail. It is an experimental fact that most subatomic particles have a property known as spin. In fact all massive non-composite particles[1] have an irreducible spin of at least ½ħ Js. The name is assigned because in some ways it is like simple rotation, and it does involve rotation. It is in fact the total intrinsic angular momentum of the body.

Ordinary linear momentum has the dimensions $[MLT^{-1}]$ but spin and angular momentum have the dimensions $[ML^2T^{-1}]$. Also, ordinary rotation involves some body of finite size more or less rigidly spinning on an axis, and this gives it the property of angular momentum. Subatomic spin however is not quite so simple[2]. This is why we refer to it as "spin angular momentum" rather than just calling it "angular momentum". Basically, unlike simple rotation, spin is more like a simultaneous rotation around **two** orthogonal axes.

We already have a model of the naked electron as a black hole, that is, a mass enclosed within an abstract mathematical surface that arises as a consequence of the physics of escape velocity. In the case of a black hole, the escape velocity at the surface is the speed of light, and it is meaningless to consider any surface closer to the centre because that would imply a velocity of escape, or arrival from infinity which numerically equals escape velocity, greater than c.

General relativity tells us that time is infinitely dilated at the surface of a black hole, so we as outside observers can never see an object fall *through* an event horizon[3]. Furthermore no entity on the surface is able to rise above it, and it is meaningless to consider it moving inwards. From the point of view of an observer in real space then, all the mass of a black hole should be regarded as residing in its surface simply because it is meaningless to think of anything inside. The interior of a black hole is not a part of real space, albeit real space encloses it.

The surface should logically also be regarded as completely fluid. It is not a rigid shell after all, but just a mathematical manifold that in physical terms possesses energy, and therefore mass. The idea of assigning rigidity to the surface is in fact an unnecessary complication, and probably comes out of our experience of planets and stars possessing a degree of rigidity that requires them to act like solid bodies. This is neither required nor justified in the description of a black hole however. Such a body does not have to rotate on an axis like a rigid sphere.[4]

Nevertheless we have to regard the electron as spinning in some way. One clue is that any object falling onto a black hole from infinity must be moving at c. Furthermore, time dilation notwithstanding, such an object must still be moving at c after arrival. After all, it cannot meaningfully go any further inward from an observer's viewpoint and there is no solid surface on which to impact and expend its kinetic energy.

There is only one way we can conceive of this occurring. Every part of the black hole's spherical surface must be considered to be moving at c parallel with the surface, and unless all the velocity vectors happen to cancel out, this can represent a type of rotation.[5]

In the case of ordinary rigid rotation all the action occurs in the x-z plane, and it can be regarded as a single vector quantity measured along the rotational y axis. However if the mass of an event horizon *everywhere* moves at c, this involves the x-y and y-z planes as well and rotational vectors can be described along x and z. We do not have to consider all three planes however. We can fix our own reference frame along one of the axes, and conventionally this would be z, with a rotational plane in x-y.

This leaves just the rotational planes of x-z and y-z to consider with rotational vectors along x and y. However we cannot avoid at least one extra axis if we are to describe what amounts to a kind of tumbling action. Another spatial dimension has been introduced and it necessarily generates a second rotational vector which explains why the dimensions of spin contain an extra unit of length. In classical terms we might consider it a type of precession, but the truth of the situation is rather more complex than this.

For a subatomic black hole the process is quantised, and the degree of spin is determined by how much the surface can be thought of as revolving predominantly in the x-z plane, and how much in the y-z plane. If it were to move entirely in x-z, the particle would revolve on an axis like a wheel and have angular momentum but no spin because that physical quantity requires there to be a second rotational vector.

Furthermore, an event horizon just won't revolve this way because it isn't rigid. If however a quantum black hole surface can be thought of as revolving in x and y simultaneously such that it revolves twice in x while it rotates once in y then the surface has one unit of spin, namely $\frac{1}{2}\hbar$ Js. This is a kind of precession in y, but unlike true precession which occurs under the influence of an externally applied force, this behaviour is intrinsic. It is a quantum effect.

There is actually a symmetry involved here. We can visualise geometric systems involving the transformations of rotation, translation and reflection, that when subjected to certain sequences of these, pass through a series of conditions that eventually return them to their original states. This sequence of transformations defines the symmetry of the system.[6]

Just this kind of process is involved in the quantisation of spin. Here however, it is the amount that the surface rotates in x and y that decides the symmetry. In effect we have to ask, given a particular partitioning of the rotation into x and y, how many rotations of x in terms of y does it take to restore the system to its original state?

In the first case it takes two rotations of the x axis to one rotation of y to restore the original state of the system. The next quantum state calls for two rotations of x but three rotations of y, and the particle has $^3/_2\hbar$ Js of spin. Two rotations of x and five of y to restore symmetry give a spin of $^5/_2\hbar$ and so on. Particles that only ever step up through this quantum series in half integral units are known as fermions. Bosons, mentioned earlier, go up in integral values.

Now with an understanding of what fermion spin is in physical terms, at least for the electron, we now come to some rather far-reaching considerations. First let's consider the self-energy of the electron. This is just

$E = m_e c^2$

How much of the electron's self-energy is due to its spin? All bodies revolving around some central point store kinetic energy, expressed for a simple point mass as

$E_\theta = mr^2\omega^2$

where ω is angular velocity $d\theta/dt$.

We can thus suggest that the spin energy of the bare electron should be regarded as the sum of all energies of all infinitesimal mass elements in its surface, all revolving at a distance r_e from its centre. This simplifies to just

$E_s = m_e r_e^2 \omega^2$.

The simplicity of this solution belies the fact that an object like our black hole electron does not rotate in a simple way. As previously indicated, all points on the surface must be considered to circulate along complex paths. If we also allow that the surface is not a rigid shell, but more like a rubber sheet that can stretch and relax, then we can ensure that all points circulate along paths that carry them around at c.

A visualisation of the type of path involved might be something like the seam on a tennis ball. Since we do not require the event horizon to be a rigid structure we are quite entitled to do this. After all, it is more a mathematical limit than an actual physical thing. Mathematically, it has no thickness for example. Perhaps though this is too severe a judgement. It does after all have mass and spin, and the uncertainty principle forbids us to say it absolutely has no *physical* thickness. So perhaps it should be regarded as a physical object, albeit a very peculiar one. It also has charge in the case of the electron, and Steven Hawking has demonstrated that it even has a thermodynamic temperature and entropy[7].

But above all it does have mass and therefore intrinsic self energy. Consider this mass to be the infinite sum of all mass differentials of which it is composed. Let us also keep things simple by treating it as a sphere.[8] This means that we can simplify the problem to one of a mass consisting of the sum of all these mass points, continually moving at c around a radius r_e.

This is mathematically equivalent to a dumbbell with a massless arm and the mass concentrated into a point, a structure that does not require us to work out an awkward moment of inertia.[9] That quantity becomes just the sum $\sum r_e^2 dm_e$. We thus have a valid albeit wholly abstract mathematical model. Although it does not visually describe the actual physical situation, it works.

The dumbbell model allows us to state the angular velocity as

$\omega = c/r_e$

If we now substitute this into the spin energy equation above, we can say that

$E_s = m_e r_e^2 c^2 / r_e^2$ so

$E_s = m_e c^2$

which suggests that the self-energy of the electron derives wholly from its spin, a rather surprising result particularly as it was so easily found. One might well ask why such a simple result has not been found before? This must be answered by pointing out that it *depends* upon the model of the electron as a quantum black hole. Without that proviso there is no real motivation to explore this relationship.

In other words, the rest mass of the electron is in fact kinetic energy, and this is very exciting. It suggests that if this is the tip of a fundamental principle underlying all matter, then all positive energy is at groundbase purely the energy of motion, and this even includes rest mass energy. There is an elegant simplicity here.

So the rest mass energy of the electron is kinetic. And kinetic energy is positive in the total energy equation

$E_t = E_k + E_p$

where t is 'total', k is 'kinetic, and p is 'potential'. Note that the last term is essentially a negative energy as are all field potentials involving attraction between entities. In particular, the electron has a potential energy due to its own gravitational field, and this is a negative energy. This is essentially so for any extended body. Any mass has a gravitational self-energy derived from the weight of its own parts. It is particularly simple for a black hole though, because all the mass is effectively resident in its surface.

Imagine the earth as hollow, with all its inner matter crammed into a thin superdense shell just under your feet. You'd still be pulled down with the same weight because a mass always acts as if it were concentrated at its centre of gravity which for regular bodies is the geometric centre. For our hollow world, the centre of gravity is its true centre and the shell would be a mass M pulled downward by a mass M.

Certainly there is only one mass involved, but the hollow body situation behaves as if there were two. We may thus say that for the electron, (note the negative sign)

$-E_p = G_e m_e^2/r_e$

$= G_e m_e^2 c^2/(G_e m_e)$

$= m_e c^2$

This says that the electron possesses a gravitational negative potential energy component that cannot be seen, but that its opposite can. This invisible energy apparently manifests in a positive sense as visible self-energy equal and in fact equivalent to the electron's positive rest mass energy. So it seems that not only is the electron's rest mass energy the kinetic energy of its surface, but that it is also the gravitational potential of that surface in positive terms.

This seems somewhat surprising. How does an object exist if its self-energy is partitioned into equal and opposite parts? Surely the total energy would be zero and the object could not exist? This is true if we restrict ourselves to positive energy space alone. However, when we recognise that the mass-energy equation has two roots, and that this implies that there is a real negative energy space, the situation makes sense. The two energies can each reside in its own space and the object may exist.

This was first recognised by P.A.M. Dirac in the nineteen twenties.[10] He postulated the existence of an all-pervasive negative energy space and from it he predicted the existence of antimatter as an unavoidable consequence. However Dirac's view may well have been somewhat naive at the time. If we regard negative space as wholly manifest in the potential energies of *real* space, we don't have to contend with the idea of some spooky nether world of invisible entities, or as was suggested at the time by some, 'mirror worlds'! Instead, the total energy of all matter, and of the universe itself is in fact zero but bifurcated into positive (kinetic) energy and negative (attractive field) energy.

The present paradigm of initial creation is that the universe arose as a quantum fluctuation of the vacuum, followed by a vast inflationary expansion.[11] Reality is seen as arising ex nihilo by a bifurcation of zero energy. It divides into a positive kinetic part, which includes all matter and motion, and a negative potential part which incorporates all attractive fields, and the gravity field in particular.

All action then becomes the consequence of interplay between these two regimes – the process by which one converts to the other conservatively. We are familiar with the idea of potential energy converting to kinetic energy and vice versa. The idea is that this is actually the underlying basis of all action.

Furthermore, treating positive mass energy - kinetic energy and matter - as an abstract energy space, and likewise the attractive fields as a negative energy space, then all action occurs at the interface between the two. It is an observable fact that the most complex activities of nature inevitably occur at interfaces, and the complexity of basic reality would appear to operate in the same way. This is a deeply satisfying conclusion as it implies an elegant simplicity underlying all existence.

More important from our immediate point of view though, is to find that for the electron at least, its positive real energy is precisely balanced by its gravitational self potential. Its total energy is actually zero which is as it should be according to such an interpretation as that described above. If we couple both positive and negative space together then the total energy of everything is truly zero.

But have we considered every source of energy? We must also note that because $G_e m_e^2 = \gamma e^2$ we only have to substitute the electrical term for the gravitational term in the above equation to show that the electrical potential of the electron's surface is

$$-E_\gamma = \gamma e^2/r_e$$

$$= G_e m_e^2 c^2/G_e m_e$$

$= m_e c^2$

yet again.

But care - the above equation was originally formulated for two *like* charges trying to push each other apart. Negative electrical potential surely applies to unlike charges trying to fall towards each other in the same fashion as two masses? For the electron as a single body we have to regard it as trying to blow itself apart, but its enormous (for its size) gravity field precisely balances this tendency. Doesn't this imply a *positive* electrical potential?

Indeed it does. Note though that if we restate the above as

$E_\gamma = \gamma e^2/r_e$

$= G_e m_e^2 c^2/G_e m_e$

$= m_e c^2$

the electrical potential is in fact positive as it should be. And the likeness or unlikeness of the charges simply loses significance when the term e is squared. What we have just done is to show that for the electron its rest mass energy, its electrical self potential and its spin energy are the same thing. And that they precisely balance the gravitational potential energy under $G_{e.}$

In truth then

$-G_e m_e^2/re = m_e r_e^2 \omega^2$ (inverse gravitational potential energy equals spin energy)

$= \gamma e^2/r_e$ (equals electrical potential energy)

$= m_e c^2$ (equals rest mass energy)

This is a very profound result. If it could be shown that all matter is fundamentally leptonic, and that the electron itself is the basis of all matter, then this concept could be extended to all mass in a general sense. Certainly quarks, the constituents of hadrons, can convert to electrons as in neutron decay, but we know too little of the mechanics to say that quarks are simply electrons involved in some super-energetic sideplay that gives us the hadrons. A nice idea, but….

References and notes;

1. All subatomic quanta fall into two broad categories; *bosons* with integral spin values, and *fermions* with half integral spin values. All fermions except the neutrino (possibly) have rest mass and are regarded as truly elementary. That is, they are non-composite. Some bosons are elementary, and others are composite. The photon is an elementary boson, but the pion is not. It consists of two quarks, which are elementary (and fermions). All composite bosons have rest mass. Some

non-composite ones do too, but the photon is a non-composite boson without rest mass. From this somewhat tangled grouping, it can be seen that all non-composite particles with mass must be fermions. For more about the particle "zoo" see

Arthur Beiser; "Concepts of Modern Physics", 5^{th} edition, McGraw-Hill, 1995.

2. A lucid and non-mathematical description of the nature of spin is provided in

S. W. Hawking, "A Short History of Time", Bantam Books, 1988.

3. Given that time is infinitely dilated at the surface, not only can nothing ever be seen to pass through the event horizon inwards, a body moving outwards takes forever to disengage itself from the surface. An object not actually *on* the surface however, even if only microscopically above it could conceivably be seen to rise, given enough observer's time. But of course it would then fall back eventually if its speed was anything less than c. The following are sources that provide general information about black holes;

R. Ruffini & J.A. Wheeler; "Introducing the Black Hole", Physics Today, 24:30-36, 1971.

M.A. Markov; "The physical effects in the gravitational field of black holes", edited and translated by Kevin S. Hendzel, Nova Science Publishers, 1987.

I.G. Moss; "Quantum theory, black holes and inflation", 1996.

G.E. Tauber; "Relativity: from Einstein to black holes", 1998.

4. Despite the statement made in the text, a lot of work has been done on the physics of rotating stellar black holes. These are thought to come into being during a supernova explosion when the core of a star collapses. This forms a small dense body that reaches the status of a black hole when its surface becomes coincident with an event horizon.

Mathematically it is possible to follow the collapsing mass inward to a singularity, but this has no real meaning in physical terms. More to the point, because the star is a more or less rigid body it rotates about an axis without a lot of differential movement between its inner layers or its latitudinal zones. It is actually gaseous throughout of course but the high pressure of the interior ensures that the viscosity of its substance ensures that it behaves like a semi-solid.

Classical physics tells us that as the stellar core collapses, conservation of angular momentum ensures that it will spin up to a high rate of revolution. Stars that do not quite make it to black hole status exemplify this. Spinars, or rapidly rotating neutron stars are well known. It thus seems perfectly reasonable that a heavier star that collapses to the point where its surface becomes enshrouded by an event horizon should also spin rapidly on an axis.

This might be so for the surface that is mathematically regarded as continuing to descend faster than light towards the central singularity, but in real terms the star has left real space, leaving its mass behind in the event horizon, but that is *not* the stellar surface. The stellar surface simply no longer exists, and there is no mathematical or physical requirement that the event horizon spin on an axis or be a rigid surface.

It must of course retain the angular momentum of the collapsed star, but it can do that in many ways. Rotation on an axis is only one alternative, and for a surface as fluid and abstract as this, the simplest physical solution is for there to be an equipartition of energy. Every point on the surface should move with the same kinetic energy but in such a way as to retain the original angular momentum.

Despite the physical simplicity of this it is a formidable mathematical problem to describe, and fortunately we do not have to do it. Because we are concerned with quantum black holes and not stars, the theory underlying this process is already encapsulated in the description particle spin.

The definitive work on rotating stellar (Kerr) black holes is

S. Chandrasekhar, "The mathematical theory of black holes", Oxford University Press, 1992.

5. Note that this is quite unlike a rigid rotation, where the velocities of all points depend on their latitudinal position. At the poles of rotation, the velocity is zero, and the highest speed occurs at the equator. The average overall speed however is considerably less. For a spherical surface to have every point on it moving at the same speed however calls for a special kind of locus whereby each point has to not only move with an 'equatorial' component, but it must also turn upwards and downwards to achieve a 'polar' component as well. One curve that could accomplish this might look something like the seam on a tennis ball.

   However such a locus, if symmetrical, would not provide a nett rotational character to the motion, as all rotational aspects of the locus are balanced by others of opposite sense. There are however other curves that might achieve constant velocity everywhere but allow for a nett rotation. On the other hand, the nature of spin suggests that whatever their form, the loci are probably not symmetrical, and that they can switch to a greater bias in one plane than another as quantum factors dictate.

6. As examples, consider a hexagon with each sixty degree segment painted a different colour. This would be 'invariant' - that is it would return to its original state - after six rotational transformations of sixty degrees, or one of three hundred and sixty. The shape and orientation of a kite are invariant under two reflections through the short axis. Through the long axis they are invariant under reflection in general.

   Invariance can apply to a single property, thus for example, shape is invariant under enlargement. Also transformations can operate between geometries. Transformation from a Euclidean to a non Euclidean geometry preserves areas but not angles.

The familiar Mercator projection of a sphere onto a flat map (one of the reasons a transformation is frequently described as a mapping) fails to preserve area, shape and distance. Greenland looks as big as Africa and is distorted. A better projection is the equal area projection that preserves areas and distances but does severe violence to shape particularly near the edges of the map.

And many transformations are quite abstract, such as the Lorentz-Fitzgerald transformations. Many physical processes are deeply concerned with invariants under various transformations. That is, we seek to find underlying symmetries in physical processes in order to discover natural laws. The law of conservation of energy for example is a symmetrical invariance under the transformation of time reversal, and that is *very* abstract!

There are a great many works on transformation in many disciplines. Here is a representative sampling;

A general geometric approach to transformation and symmetry is

M. G. Edward, "Transformation geometry: an introduction to symmetry", Springer-Verlag, c1982.

Rather more advanced is

P. J. Kelly and E. G. Straus, "Elements of analytic geometry and linear transformations", Foresman Scott, 1970.

A more abstract algebraic approach is taken in

B.H. Bunch, "Reality's mirror: exploring the mathematics of symmetry", Wiley, c1989.

C. G. Cullen, "Matrices and linear transformations", 2nd edition, Dover, 1990.

Transformation applied to physical problems is described in

J. P. Elliott and P. G. Dawber, "Symmetry in physics", Oxford University Press, c1979.

C. J. Tranter, "Integral transforms in mathematical physics", London, Methuen; New York, Wiley, 1956.

A highly authoritative work is

G. 't Hooft, "Under the spell of the gauge principle", World Scientific Publishing, 1994.

A very recent compilation is

A. Faessler, T.S. Kosmas, G.K. Leontaris, eds. "Symmetries in intermediate and high energy physics", Springer, c2000.

7. S. W. Hawking, "A Short History of Time", Bantam Books, 1988

8. This might seem an oversimplification given that the electron has rotational properties, and this might therefore be expected to introduce geometric distortion due to inertial forces. However, if we consider a mass revolving about an axis on the end of an elastic arm (this is an equivalent problem) without any impulsive torque, then no matter how the arm is lengthened, the angular momentum of the system must be conserved.

   Consequently the velocity of the mass reduces accordingly if the arm is stretched by inertial force. However the total energy of the system must also be conserved. The kinetic energy of the mass is reduced, but the strain on the elastic arm increases, and this is a form of potential energy. The kinetic energy plus this potential remains constant. The system will take up whatever configuration satisfies both these conditions.

   If all we are concerned with is the total energy of the system there is no need to correct for geometric distortion. Assuming a sphere with surface velocity c, this allows that if the electron distorts outwards, this velocity will decrease which is quite permissible, but the total energy will be conserved, and that is all that counts. This matter is so elementary that the reader is invited to work it out if it still remains a worry.

9. A full description of the kinetics of rotating systems and methods for calculating moments of inertia can be found in any undergraduate physics text on mechanics and any comprehensive text on the integral calculus.

10. $E^2 = m_o^2c^4 + p^2c^2$

    Solves to

    $E = mc^2$ but also to $E = -mc^2$

    Einstein himself rejected the second result on the grounds that negative mass is not observed, but Dirac was less didactic and allowed that though mass is always observed to be positive, energy itself could be negative. In 1928 he postulated a negative energy space (which was more a mathematical space than a physical one) which by its nature predicted the existence of anti particles. This prediction was born out four years later with the discovery of the positron.

    In his original model, he assumed that because energy always seeks its lowest level, this space was bottomless and full. However this is not the only interpretation. If instead one suggests that energy tends to zero, then negative energies tend upward while positive energies tend downwards and energy space is symmetric and totals to zero.

    P.A.M. Dirac; "The principles of quantum mechanics", 4th edition, Clarendon Press, 1958.

11. J. D. Barrow, "The left hand of creation: the origin and evolution of the expanding universe", Oxford University Press, 1994.

E. Vanmarcke, "Quantum origins of cosmic structure", Rotterdam; Brookfield, VT: A.A. Balkema, 1997.

A.D. Linde, "Inflation and quantum cosmology ", edited and with a foreword by Robert Brandenberger, Academic Press, c1990.