The Gravitational Electron

Another derived constant from the era of quantum mechanics is the 'classical radius of the electron'. This object arose as a necessary adjustment to quantum electrodynamics when it was still very young. In the early days of QED some problems had an unfortunate tendency to blow out into unworkable infinities.

The trouble lay in assuming the electron to be a truly pointlike particle which suggested that its own self energy was infinite, and this is obviously not so. The classical notion simply didn't work, but by assigning the electron a tiny finite radius, this problem of infinite corrections dissolved[1]. Very simply, the classical radius is

$$r_e = \gamma e^2/m_e c^2$$

$$=2.817939 \times 10^{-15}\text{m}.$$

Also, given that

$$\gamma e^2 = G_e m_e^2$$

It follows that

$$r_e = G_e m_e^2/m_e c^2$$

$$r_e = G_e m_e/c^2$$

For present purposes this equation is best regarded as describing the radius of the electron at the time when the infinite fields can be though of as equivalent[2]. Its main significance is that it describes the conditions to which the electron was subject shortly after the creation of the universe. Note that this does *not* imply that it describes the radius of the electron today.

Usually the electron is regarded as a particle equal in size to the wavelength of light used to reveal it most efficiently, but the uncertainty principle determines that all we are likely to see is a relatively large object. The special wavelength that can illuminate this entity effectively is known as the Compton wavelength of the electron[3], and it is $1/\alpha$ (about 137) times larger than the classical radius. Its usual definition is

$$\lambda_e = \hbar/m_e c$$

$$= 2.4263096 \times 10^{-12}\text{m}.$$

One interpretation of this is to treat the electron as a small hard core or *bare* form, flitting about inside a larger space due to quantum fluctuation[4]. It is this larger space we see when we attempt to illuminate the elusive particle itself and we can regard this as the *clothed* form of the electron. It must be realised that the Compton wavelength is itself a fundamental limit of measurement and the electron can never be resolved any more finely using light as a probe.

A beam of ultra high-energy particles can however break through this limit to some extent depending on their mass-energy and thus their effective wavelength. However despite experimental efforts to probe the interior of the electron, mainly with the Stanford linear accelerator at Brookhaven[5], nothing like a hard core has ever been detected. There is a reason for this however which will become apparent shortly.

In particular, for the purposes of this discussion, considered as a relativistic statement, the above equation declares that there once existed a free naked form of the electron. It had a radius r_e *and this was the radius of the event horizon of a quantum black hole*. This statement is justified because the Schwarzschild radius[6] commonly quoted to define a black hole is

$$R = 2GM/c^2$$

It must be realised however that Schwarzschild himself first stated this relation in 1917 in a paper designed to demonstrate that Einstein's (then) new general theory of relativity necessarily implied an inner bound or horizon to the universe. This was in addition to the outer bound (now identified with the Hubble limit) already stated by Einstein himself[7]. Schwarzschild did not actually identify his inner horizon with real stellar objects however.

Instead it was identified with the now defunct idea of a small 'cosmic egg' from which the universe was hatched. Old ideas do look very quaint in the light of the wisdom of hindsight. Indeed it was all very abstract. In fact it was to be half a century before stellar black holes as such became an acceptable proposition. It was J.A. Wheeler in 1969 who first coined the term 'black hole' to describe such objects as real entities in cosmic space[8].

In his exposition, Schwarzschild actually provided the first exact solution to the Einstein field equations. Of course too, his paper was basically a critique of one aspect of the new theory. But he was using a very simple model that did not envisage that the limiting horizon might actually possess spin. The Schwarzschild metric is fundamentally radial and static.

However of course, the electron does in fact spin, with an angular momentum of $\frac{1}{2}\hbar$ Joule seconds. The Schwarzschild metric is thus not really suitable for the description of an electron as a quantum black hole, and instead a Kerr-Newman[9].metric is more appropriate. In simple terms, for an electron, the coefficient 2 in the Schwarzschild model cancels against the half integral spin coefficient to yield an event horizon at

$$R = GM/c^2$$

and this surely means that the naked electron was created out of the primitive G_e field as a quantum black hole.

A very important proviso has to be dropped in at this point. Steven

Hawking has shown that black holes evaporate due to pair production close to their event horizons[10]. This process accelerates until the hole finally expires in an explosive burst of energy. The electron has to do this too, or the proposition fails. But the electron is completely stable.

Or is it? Consider – an electron generates a polarised virtual electron-positron pair close to its event horizon, borrowing energy from the vacuum within the constraints set by the uncertainty principle. This is simply the normal quantum fluctuation of the vacuum, but in the proximity of a charged particle this virtual matter field becomes polarized. The positron moves toward the electron, while the virtual electron moves away. The extant electron then combines with the positron and is annihilated. Meanwhile the virtual electron effectively gets away.

This newly generated electron however can only move as far as a single Compton wavelength before the process must end either by repaying the energy debt or forcing the electron to sink back into the negative energy space from which it arose[11]. However the original electron is annihilated together with the positron, so the energy debt to the vacuum is repaid and the new electron can materialise. In the end, because all electrons are identical, the electron appears to have simply moved in a sudden quantum jump. Then it does it again. And again. This is quantum fluctuation, and it is completely adequate to satisfy the Hawking requirement. Of interest, only an electron could get away with this.

The reason is that more massive particles are permitted to decay into entities smaller than themselves. Not so the electron because there are no smaller entities that would conserve the essential properties of mass, charge and spin. The most important inference that can be made from this is that not only was the electron created as a quantum black hole when G was G_e, but because the electrons are still extant and have not evaporated away as we might expect, then they are quantum black holes *today.*

Now, it is a very radical statement to identify the electron with a gravitational black hole and it is worthy of a double check. In the following we will *not* take as a premise that gravity varies with time. Instead we will use quantities observable today and use them to show that not only must gravity vary, but that the electron necessarily was once a black hole.

The argument is somewhat more convoluted, but it is necessary to pin down these assertions firmly as undeniable facts under the terms of this opus. The classical radius of the electron is

$$r_e = \gamma e^2/m_e c^2$$

and the fine structure constant is

$$\alpha = \gamma e^2/\hbar c$$

Resolving both for γe^2 and equating, we get

$$r_e m_e c^2 = \hbar c \alpha$$

which becomes

$r_e = \hbar c\alpha/(m_e c^{2)}$

Dividing through by m_e gives

$r_e/m_e = \hbar c\alpha/(m_e^2 c^2)$

and multiplying through by c^2 leaves

$r_e c^2/m_e = \hbar c\alpha/m_e^2$

which we recognise as a constant and we will call it G_e. Dimensional analysis

shows that this is just a variant of G.

This means that

$r_e c^2/m_e = G_e$ and

$r_e = G_e m_e/c^2$

This is the black hole equation yet again and the presence of the object G_e, dimensionally equivalent to G, casts grave doubt on the constancy of the latter.

There is yet another independent way to arrive at the black hole statement, but first we need to acquaint ourselves with the Planck units[12]. These rather arcane structures were developed by Max Planck in an effort to create a set of natural units that would be independent of any local or arbitrary units, and even of the existence of humanity itself.

He combined the gravity constant with the two central constants of electromagnetism, the speed of light and the constant that was to bear his name. Also because mass and charge appear as squared terms in the force laws, the units were better reduced to their square roots. This way they enter those laws as linear terms.

To an extent, the units were clever concoctions, but in fact they work with astounding veracity. Today, though still seen as rather enigmatic entities, they are regarded as setting the absolute limits of meaningful measurement, at least for length and time given that these are extremely small quantities. This view however must be suspect because it simply doesn't work for mass. The mass unit is actually rather large.

Anyway, the units are

$L = (G\hbar/c^3)^{1/2}$

$T = (G\hbar/c^5)^{1/2}$

$M = (\hbar c/G)^{1/2}$

If G_e is substituted into these then

$L_e = (G_e\hbar/c^3)^{1/2}$

$T_e = (G_e\hbar/c^5)^{1/2}$

$M_e = (\hbar c/G_e)^{1/2}$

so these represent the Planck units when G was G_e. Obviously if G varies, so too must the Planck units, and it is easy to see that then they were a lot larger than they are today, though the mass unit was a great deal smaller. We might suppose that although the mass unit means little today, it might have had some significance as a limit of measurement then.

We can manipulate these now to give

$L_e^2 = (cT_e)^2$

$= G_e\hbar/c^3$

$= G_e m_e/c^2 \times \hbar c^2/m_e c^3$

$= r_e\hbar/m_e c$

$= r_e\lambda_e$

where λ_e is the electron Compton wavelength again. But

$\lambda_e = r_e\alpha$ so

$L_e^2 = r_e^2\alpha$

We can also perform similar manipulations on the unit of time to reveal that

$T_e^2 = r_e^2/c^2\alpha$

Or if we care to invent a unit t_e we can call 'electron time' such that

$t_e = r_e/c$ then

$T_e^2 = t_e^2/\alpha$

Likewise the mass unit reveals that

$M_e^2 = m_e^2/\alpha$

These inferences are easily checked by substitution, and it is seen that there is thus a simple relationship between the limiting units as defined by Max Planck and the more familiar natural constants.

If one now invokes the property of spin, given that the electron has an irreducible spin of $\frac{1}{2}\hbar$ Js, let

$$\tfrac{1}{2}\hbar = m_e c^2 T_e/(2\alpha^{1/2})$$

The reader is advised to check this by substitution. If we square this to get rid of the square root and also expand T_e then

$$(1/4)\hbar^2 = m_e^2 c^4 G_e \hbar/(4c^5\alpha)$$

and dividing through by $\frac{1}{2}\hbar$ gives

$$\tfrac{1}{2}\hbar = m_e^2 c^4 G_e/(2c^5\alpha)$$

and this gives us a time dependent form of the spin quantum in gravitational terms.

But the spin quantum can also be stated in a length dependent form

$$\tfrac{1}{2}\hbar = m_e c r_e/2\alpha$$

You are invited to check the validity of these relations by substituting values and analysing the dimensions. Combining the two forms,

$$m_e c r_e/2\alpha = m_e^2 c^4 G_e/(2c^5\alpha)$$

Canceling through to remove common terms and rearranging we find that

$$m_e c^2 = G_e m_e^2/r_e$$

$$c^2 = G_e m_e/r_e$$

and this is just the black hole form yet again.

These derivations are all essentially independent of each other but all arrive at the same conclusion. The naked electron appears to have the properties of a quantum black hole that was created with a radius r_e when G was G_e and this black hole is still extant. It suggests very strongly that we have to concede the status of the classical radius of the electron. It is or was a genuine event horizon.

However we need to careful how we interpret r_e. Since

$$r = Gm/c^2$$

describes a general black hole, and since c is a constant, if m is also constant, then if G decreases then r must do so too. This implies that today from our viewpoint, the event horizon of the electron must be regarded as having shrunk. Its modern value must be

$$r = r_e\, G/G_e$$

However a very important proviso needs to be made. It is a property of a black hole that due to the effects of general relativity[7], time is infinitely dilated at the event horizon.[13] This means that in the reference frame of the electron itself, time at its surface has not progressed since it was created.

To the electron, G is still Ge and it is still at the moment of its own creation. This is because a black hole is just its surface and nothing more. It is in fact meaningless to think of anything inside a black hole because the interior is not part of real space. From the viewpoint of any outside observer, the mass of a black hole resides entirely within its surface.

This also serves to explain why scattering experiments have never revealed a hard kernel to the electron. For one thing, any incident particle would be trapped in the electron's gravity well. No incident particle that was on target could possibly bounce back. Even at G modern, the black hole though very small is still perfectly capable of trapping anything that descends to its surface. In this respect all black holes are alike no matter how small.

And the electron core is indeed very small. From our point of view, in our reference frame, r is only 10^{-58} metres. Essentially this is a pointlike object. Even if an incident particle *could* scatter off it, no hard core of measurable size would ever be observed.

References and notes;

1. Initially, applying the classical radius of the electron as a correction factor to problems generating infinites was not entirely satisfactory. Although it served to provide correct numerical results the scheme could not be made entirely self consistent, even though it seemed to work in practical terms.

   In situations involving electrons in relativistic motion, corrections had to be applied to the mass and charge, but without the classical radius these became infinite. Assigning the classical radius to the electron, these corrections became equal to the true mass and charge of the electron as measured experimentally, so the adjustment was generally accepted despite its lack of strict theoretical consistency.

   Then in 1950, new renormalisation techniques were brought to bear on the problem and a partial solution was found. Even so, the classical radius was not seriously regarded as the radius of some hard kernel inside the larger charge cloud of the electromagnetic electron. To many it still seemed to be a kind of fudge that was not properly justified.

   The trouble is that all attempts to experimentally probe the electron using scattering techniques, completely fail to find any finite kernel within, and on principle the bare electron cannot ever be observed in electromagnetic terms.

Because of these limitations, many workers still regard the electron as a vanishingly small object immersed in an ever more dense charge cloud as one approaches a point singularity in the middle. Whatever the physical reality however, in theoretical terms, regarding the correction factors of the mass and charge as the electron's actual mass and charge works fine and the theory is highly predictive. For a good historical account see

D.L. Anderson; "The Discovery of the Electron", 1964.

P.A.M. Dirac; "Quantum electrodynamics", Dublin Institute for Advanced Studies, 1943.

P.A.M. Dirac; "The principles of quantum mechanics", 4th edition, Clarendon Press, 1958.

2. In this opus, the classical radius is in fact being treated as a true kernel of the observable electron, and also it is to be regarded as a quantum black hole. As such, no scattering experiment can ever reveal it, simply because anything fired at a black hole *never comes back*!

   Furthermore, if the bare electron is a black hole, it is a gravitational body, and furthermore its radius is defined in terms of G, and a variable G at that. It is therefore not surprising that the theory to date has not been made entirely self consistent because a vital piece of data is missing. That it works despite this however is because the value of r_e is actually numerically correct so it gives the right answers!

   Note though that the expression for r_e is defined as the radius of this black hole when the G was about 10^{32} units. Since then G has declined to 10^{-11} units, and it follows from our point of view that the event horizon of such an object must be reduced by the same amount since c and m_e are constants. It follows that if r_e is about 10^{-15} metres, then the modern radius of the bare electron is only10^{-58} metres, which is about as pointlike as one could ever wish for!

3. The Compton wavelength is determined experimentally by scattering x-ray photons off electrons in a metal (where there are plenty that are essentially free.) Classically, the Compton effect was a demonstration of the quantum nature of light, but it also secondarily provides a measure of the size of the charge cloud (the quantum fluctuating polarised population of virtual particles) associated with the electron.

   The Compton wavelength appears in the scattering equation as a constant that is independent of the incident wavelength, and functions as the size of the object that is scattering the light. By analogy with wave behaviour in general, the greatest energy exchange by scattering tends to occur when the incident light has a wavelength that is the same size as this object.

   If in doubt, consider an aircraft carrier at sea. Do the waves affect the ship appreciably? Likewise a cork just bobs up and down, but put a two metre rowboat out in a two metre chop, and both the boat and the waves are strongly affected.

Another way to think of this quantity is to consider the quantum uncertainty of the electromagnetic radius of an electron from its mass energy and the speed of light. The Heisenberg uncertainty principle states that for an electron

$\Delta m_e c^2 \Delta t = \hbar$

$\Delta m_e c \Delta ct = \hbar$

$\Delta m_e c \Delta r = \hbar$

Eliminating the deltas by treating the terms as total integrals over their domains

$r = \hbar / m_e c$

and this is the Compton wavelength λ_e.

In effect it says that we cannot hope to resolve the electron any more finely than this by using light. It also establishes a size for the clothed or electromagnetic electron. A treatment of the Compton effect can be found in any undergraduate physics text.

4. Quantum fluctuation is a state of activity associated with any particle due to its uncertainty in position and momentum according to Heisenberg's uncertainty principle. Given any particle of mass m, it can disappear entirely and reappear somewhere else completely at random provided it does so within a time t, and within a distance r of its original location as defined by the equations

   $m c^2 t = \hbar$

   $m c r = \hbar$

   Even the physical vacuum quantum fluctuates because the uncertainty principle allows any object of mass m to spontaneously appear together with its antiparticle, in effect establishing an energy debt to the vacuum.

   Uncertainty of energy and time however only permit this transaction provided the pair then disappears again within time t, repaying the debt, and of course its components cannot travel further than r = ct. This quantum fluctuation of the vacuum is an ongoing process that allows 'virtual' quanta of all types and sizes to be continually created and destroyed, and this gives the vacuum an irreducible "zero point energy" that can be experimentally detected.

   This relies on the Casimir effect, first predicted by Hendrik Casimir in 1948

and later confirmed by M.J. Spaarnay at the Philips Laboratories, Eindhoven in 1957. Also Dewitt 1975 and 1989, and Milonni, Cook and Coggin in 1988. A recent high resolution confirmation of the effect was recently performed by

S. Lamoreaux; Physical Review Letters, volume 78, Page 5, 1997.

In addition there are many fine works on quantum mechanics that can be consulted to expand on the above ideas. The two most definitive works are

W. Heisenberg; "The Physical Principles of Quantum Theory", 1930.

P.A.M. Dirac; "The Principles of Quantum Mechanics" 4th edition, 1958.

5. Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies; available from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Va., 1968

6. Misner Thorne and Wheeler, "Gravitation", 1973. Chapter 31.

   Schwarzschild's original description of the event horizon can be found in the following collection;

   K. Schwarzschild, "Original papers from 1887 - 1916", compiled by Max Born and AIP-ICOS, and held by Universität Gottingen, Universitätarchiv. Copied and held on microfilm at the American Institute of Physics, Niels Bohr Library.

7. A. Einstein; "Die Grundlagen der allgermeinen Relativitätstheorie" (The Foundations of the General Theory of Relativity), 1916.

8. R. Ruffini & J.A. Wheeler; "Introducing the Black Hole", Physics Today, 24:30-36, 1971

9. Misner Thorne and Wheeler, "Gravitation", 1973. Chapter 33.

   A somewhat simplistic argument can also be made as follows;

   The total energy of a body is

   $E(t) = mc^2$

   and its kinetic energy is

   $E(k) = (m - m_o)c^2$

   The total energy of motion is thus

   $E(t) = m_oc^2 + (m - m_o)c^2$

The gravitational potential energy is

$E(p) = - G m_o M / r$

but if the body is moving we must state this as

$E(p) = - G m_o M / (r (1 - v^2/ c^2)^{1/2})$

An escape mode requires that throughout its upward rise a mass m must have kinetic and potential components throughout its rise such that

$E(k) + E(p) = 0$ because as r approaches infinity, total energy approaches zero, so

$E(k) = -E(p)$. Therefore

$(m - m_o)c^2 = Gm_o M / (r (1 - v^2/ c^2)^{1/2})$

describes the relativistic situation. Then, resolving for r,

$r = (GM/ c^2) m_o/[(m - m_o) (1 - v^2/ c^2)^{1/2}]$

This will agree with our assertion if the term in square brackets reduces to m_o when v = c. We cannot simply substitute here though without committing the fatal error of division by zero. However we can substitute

$m = m_o/ (1 - v^2/ c^2)^{1/2}$

The bracketed term then becomes

$(m_o / (1 - v^2/ c^2)^{1/2} - m_o) (1 - v^2/ c^2)^{1/2}$

$= m_o(1 - (1 - v^2/ c^2)^{1/2})$

$= m_o$ when v = c.

so the relativistic form of the event horizon equation dispenses with the coefficient 2.

10. S. W. Hawking, "A Short History of Time", Bantam Books, 1988.

11. P.A.M. Dirac; "The principles of quantum mechanics", 4th edition, Clarendon Press, 1958.

12. Max Planck originally introduced his novel units during an address to the Prussian Academy on the thermodynamics of radiation in 1899;

Max Planck; "Uber irreversible Strahlungsvorgange Sitzungsberichte der PreuBischen", Akademie der Wissenschaften, volume 5, 479, 1899.

His was not the first attempt to generate a set of natural units. G.J. Stoney in 1874, addressed the British Association and subsequently published;

G.J. Stoney; "On the Physical Units of Nature", The Philosophical Magazine, May 1881.

13. M.A. Markov; "The physical effects in the gravitational field of black holes", edited and translated by Kevin S. Hendzel, Nova Science Publishers, 1987.